AD-A032 864 LOCKHEED MISSILES AND SPACE CO INC PALO ALTO CALIF PA--ETC F/G 13/7
ENGINEERING ANALYSIS USING ARBITRARY GRID FINITE DIFFERENCE TEC--ETC(U)
AUG 76 P S JENSEN, F A BROGAN F44620-71-C-0109
UNCLASSIFIED LMSC/D507700 AFOSR-TR-76-1302 NL

| OF |

AD
A032864

END
DATE
FILMED
1 - 77

(2)

ADA032864

AFOSR - TR - 76 - 1302

COPY AVAILABLE TO DDC DOES NOT PERMIT FULLY LEGIBLE PRODUCTION

D D C
RECEIVED
NOV 26 1976
B

Approved for public release;
distribution unlimited.

LOCKHEED
MISSILES & SPACE COMPANY, INC. • SUNNYVALE, CALIFORNIA
A SUBSIDIARY OF LOCKHEED AIRCRAFT CORPORATION

Approved ... release;
distribution unlimited.

ENGINEERING ANALYSIS USING ARBITRARY GRID FINITE DIFFERENCE TECHNIQUES

Paul S. Jensen
Frank A. Brogan

(14) LMSC/D507700 3 August 1976

Structures Laboratory
Lockheed Palo Alto Research Laboratory
3251 Hanover Street
Palo Alto, California 94304

FOREWORD

In this report, several experiments on the analysis of shell structures using the STAGS-FIDAG [5] computer program combination are described. STAGS (STructural Analysis of General Shells) [1] is a general engineering analysis computer program particularly useful for nonlinear problems and FIDAG (FInite Differences on Arbitrary Grids) [5] is a general program for finite difference interpolation on arbitrary grids.

The work reported here was supported by the Air Force Office of Scientific Research under contract F44620-71-C-0109, with Lt. Col. Enrique H. Ramirez contract manager.

CONTENTS

Section 1
INTRODUCTION

The large scale engineering analysis program STAGS (STructural Analysis of General Shells) [1] solves linear and nonlinear shell problems by a variational finite difference technique. It is a widely used program known particularly for its very efficient operation. In response to the needs of many users, a limited variable grid capability has been incorporated in STAGS to treat shell problems involving rather complex boundaries. The extension of this capability for more general variable grids in finite difference analysis has been the main motivation for the development of FIDAG.

FIDAG is a computer program for general, two-dimensional functional interpolation. It is applicable to direct interpolation problems such as that of producing interpolated elevations on topological maps, potential levels in electrical fields and displacement and stress levels in structural bending tests. Its primary application, however, is currently in the solution of partial differential equations, particularly when it is advantageous to utilize non-uniform grids.

The FIDAG program has been continuously evolving over the past five years and, in its various forms, has been applied to the arbitrary grid finite difference analysis of boundary value problems using both direct and variational approaches [6], and a hybrid finite difference-element approach [7]. Although these earlier studies produced good results, the full power of the program was not realized since the present Hermite interpolation capability was not implemented. In this report we present a number of results obtained by linking FIDAG with STAGS for a more general finite difference analysis capability.

1.1 Program Structure

The advantages of maximum program modularity ar well known and need not be expounded here. When operating with a program as complex as FIDAG, it is

by far simplest to have it operate as a separate, independent program, communicating with other programs through a data base. To encourage this form of utilization, a host program GRIP (GRid Processor) for FIDAG which utilizes a simple, portable data base system for interprogram communication was developed and was used for this study.

One option in GRIP is oriented toward its use in the analysis of elliptic partial differential equations such as with the STAGS (STructural Analysis of General Shells) program. With this option, GRIP determines an astute placement of control points and integration points as well as performing the FIDAG analysis. The terms control point and integration point are defined in Section 1.2 below.

Presently, using the host program GRIP, the arbitrary grid differencing capability of FIDAG has been introduced into STAGS as illustrated in Fig. 1.1.

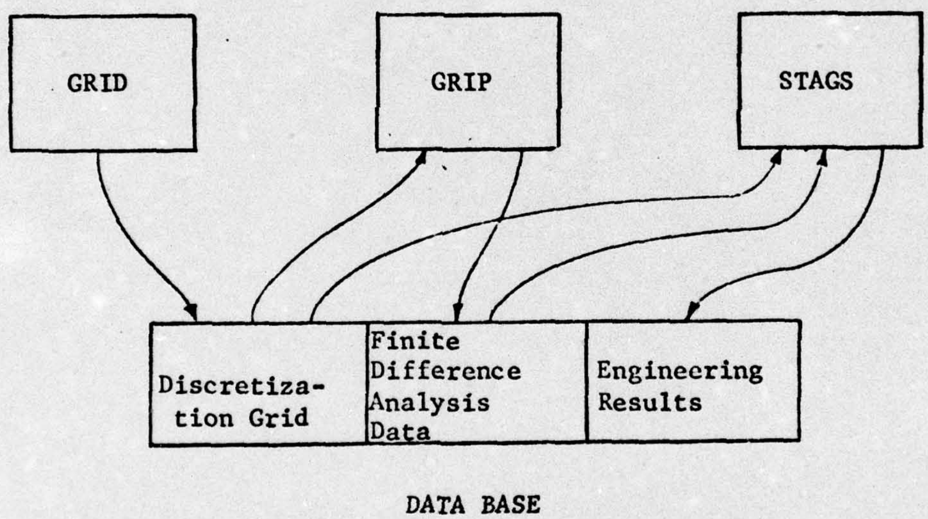


Fig. 1.1 Operation of STAGS-FIDAG System Using Three Independent Processors with Data Base Communication

The grid generator program GRID pictured is currently in a rather limited form adequate for test purposes.

There are a number of very compelling advantages to this organization including:

- individual processors can be modified with minimal side effects on other processors,
- other related processors such as load distribution and plotting programs can be added with relative ease,
- efficiency in program loading is realized through smaller "absolute" files for the programs and
- efficiency for multiple case studies is realized since redundant processes, such as grid generation, need not be repeated.

The efficiency gain referenced in the last item above is particularly important in typical structural analysis. The engineer generally is interested in various static, dynamic and buckling analyses for a given structure. Once a discretization grid has been formed by GRID and processed for finite difference analysis by GRIP, most of the above-mentioned analyses can be performed using STAGS alone with the information in the data base.

1.2 Some Definitions

Consider a smooth, bounded surface $\mathcal{S}$ over which a continuous, differentiable function $u(x,y)$ is defined. Let N and C be two sets of points, each covering $\mathcal{S}$ in a fairly uniform fashion. The points N' will be called the node points and C the control points. With each control point c in C, FIDAG will associate a set N_c of node points in N which are near c in some sense. Then, for each control point c, FIDAG produces a linear transformation (coefficient matrix) T_c which facilitates approximation of u, u_x, u_y, $u_{x}2$, ... u_{ym} at c in terms of values of u and/or u_x and u_y at the nodes in the neighbor set N_c by polynomial interpolation.

In the present application with STAGS, only the node points N are constructed by the grid generator GRID along with an element table analogous to that commonly used in finite element analysis systems. The element table defines a partition of $\mathcal{S}$ into quadrilaterals (called EIA's for Elementary Integration Areas in this report) having one node at each corner.

The program GRIP then places a user specified number of control points and integration points at quadrature positions in each EIA for use in the numerical integration of the energy functional by STAGS.

1.3 Overview

The remaining sections of this report provide results from a number of tests using the system described heretofore. Three basic problems are treated in order of increasing difficulty. In Section 2, rectangular plate bending and membrane displacement are analyzed using uniform rectangular grids. Bending of a clamped disc is treated in Section 3 using a non-rectangular grid. Finally, the combined effects of membrane and bending on a spherical cap are treated in Section 4 under both clamped and point support boundary conditions. General conclusions about the operation of the STAGS-FIDAG system in engineering analysis are presented in Section 5.

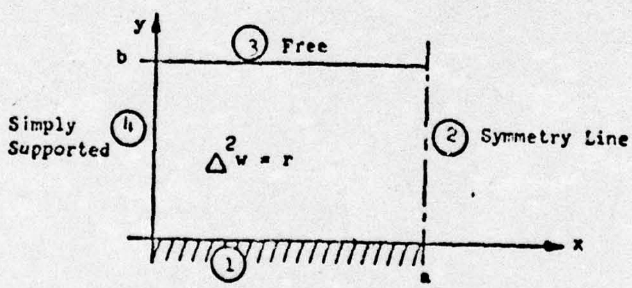
Section 2
RECTANGULAR PLATE PROBLEMS

Initial studies with the STAGS-FIDAG processor were carried out on a rectangular plate bending problem using a uniform rectangular grid. This was particularly useful for "wringing out" the system since the analytic solution is known and a great deal of comparative results are available. Earlier studies of this problem have appeared in [4, 5, 6, 7] and elsewhere.

In anticipation of the combined effects of bending and membrane behavior in the analysis of general shells, a pure membrane problem was then studied, still retaining the uniform grid. The remainder of this section is devoted to the results of these two studies.

2.1 Plate Bending Problem

The plate bending problem treated here is illustrated in Figure 2.1.



Boundary Conditions:

1	$w = 0$	$\frac{\partial w}{\partial y} = 0$
2	$\frac{\partial w}{\partial x} = 0$	$\frac{\partial^3 w}{\partial x^3} + (2 - \nu) \frac{\partial^3 w}{\partial x \partial y^2} = 0$
3	$\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} = 0$	$\frac{\partial^3 w}{\partial y^3} + (2 - \nu) \frac{\partial^3 w}{\partial x^2 \partial y} = 0$
4	$w = 0$	$\frac{\partial^2 w}{\partial x^2} = 0$

Fig. 2.1 Test Problem for Normal Displacement w under Uniform Normal Pressure

The parameters used for this analysis were:

$a = b = 0.8$ in.		
$h = 0.01$ in		Thickness
$p = 1$ p.s.i.		Normal pressure
$\nu = 0.3$		Poisson's ratio
$E = 30{,}000{,}000$		Young's modulus
$r = 0.364$		$12\, p\, (1 - \nu^2)/E\, h^3$.

The placement of the control points and integration points was made using the Gaussian quadrature placement option of GRIP. The simplest test was made using a 5 by 5 uniform grid with square EIA's. Using this grid, a number of tests were made, varying the numbers of control points and integration points per EIA and the order. The tests are identified by the form

$$C\, i \quad I\, j \quad O\, k$$

which means an i by i array of control points and a j by j array of integration points were used in each EIA, and the order of finite difference approximation was k. Thus, for example, C1 I2 03 means that one control point was placed at the center, (x_c, y_c) say, of each EIA and integration points were placed at points $(x_c - q, y_c - q)$, $(x_c - q, y_c + q)$, $(x_c + q, y_c - q)$ and $(x_c + q, y_c + q)$, where $q = 0.2886751346 \cdot e$ and e is the length of the EIA edge. Furthermore, a cubic approximation was used, which corresponds to FIDAG coefficient matrices of order $n_3 = (3+1)(3+2)/2 = 10$. The special notation $0\,k + \ell$ means that coefficient matrices of order $n_k + \ell$ were used. For example, $0\,3 + 2$ implies a coefficient matrix order $n_3 + 2 = 12$.

In finite difference analysis, constraints on the trial function space are obtained by collocation. Thus, it appears desirable to increase the support* of each local polynomial without increasing its degree. A possible approach is to combine polynomials on each EIA by weighted averaging. For example, in the case C3 I3 03 there are nine, possibly distinct cubics used on each EIA. The use of only one, derived from the given nine, is clearly a more constrained system which, however, no longer retains the exact collocation property over the entire support. Thus, the use of averaged polynomials does not necessarily produce a trial space which is a subspace of that corresponding to the non-averaged polynomials.

---

* The phrase "support of a polynomial" is interpreted as the entire set of points and function/derivative values utilized in the definition of the polynomial.

The test results corresponding to polynomial averaging, using the weights generated for the numerical quadrature integration, are identified by

$$A\,i\;I\,j\;O\,k\;,$$

where the interpretation corresponds to that for C i I j O k described previously. All of these results are summarized in Tables 2.1 - 2.4.

2.2 Observations on Plate Bending Results

In Table 2.2 we note that almost all of the absolute errors (actual - calculated) are negative. We conclude that the space of trial functions defined by the finite difference process does, in all cases, contain functions outside of the space of admissible functions which yield a lower numerical energy value than any of the admissible functions. In other words, we find that the model structure is more flexible than the physical structure.

This property is in contrast with conforming finite element analysis for which the trial space is contained in the admissible function space. Thus, the conforming element model structure is less flexible than the physical structure. This property of conforming finite element analysis holds generally whereas, unfortunately, the corresponding property of greater flexibility in finite differences does not.

The weighted mean relative errors in Table 2.4 indicate that all of the 12 freedom formulas (O 3+2 means $(3+1)\cdot(3+2)/2 + 2 = 12$ freedoms) yield about the same average error. The case A 2 I 2 O 3, which exhibited a similar mean error, indirectly utilizes 12 freedoms by averaging four 10 freedom cubics. The 12 freedoms used for these cases are values for the lateral displacement w and its partial derivatives w_x and w_y at each of the four corners of each square integration area (element or EIA) used. The best result was obtained in case C 1 I 3 O 3+2 in which one extended (12 freedom) cubic is integrated over each EIA by fifth order (3 x 3) Gaussian quadrature. We shall see later that this behavior does not extend to arbitrary quadrilateral EIA's. The weights used for the mean relative error calculations in this report were all 1.

TABLE 2,1
SQUARE PLATE BENDING PROBLEM, 5X5 GRID
CALCULATED VALUES (* 10**3)

	X=,2,Y=,8	X=,4,Y=,8	X=,6,Y=,8	X=,8,Y=,8
ACTUAL	3,51164	6,31960	8,08387	8,68127
C1 I3 O3+2	3,55722	6,41545	8,21695	8,82774
C1 I3 O4	3,71849	6,72609	8,61009	9,30803
C2 I2 O3	3,67785	6,63494	8,49927	9,13136
C2 I2 O3+2	3,55758	6,41599	8,21750	8,82829
C2 I2 O4	3,61378	6,50742	8,35407	8,96688
C3 I3 O3	3,59384	6,50276	8,33047	8,92880
C3 I3 O3+2	3,55935	6,42193	8,22522	8,83324
C3 I3 O4	3,73511	6,71804	8,57150	9,23511
A2 I2 O3	3,55953	6,41896	8,22065	8,83137
A2 I2 O4	3,58414	6,44522	8,24948	8,88024
A3 I3 O3	3,63886	6,52382	8,36673	8,95714
	X=,2,Y=,6	X=,4,Y=,6	X=,6,Y=,6	X=,8,Y=,6
ACTUAL	2,46717	4,42208	5,63946	6,04961
C1 I3 O3+2	2,49714	4,47767	5,71223	6,12838
C1 I3 O4	2,63356	4,74454	6,09515	6,52527
C2 I2 O3	2,56871	4,60738	5,87856	6,30706
C2 I2 O3+2	2,49749	4,47823	5,71286	6,12903
C2 I2 O4	2,53714	4,55042	5,80104	6,22638
C3 I3 O3	2,52110	4,52915	5,78279	6,20908
C3 I3 O3+2	2,49758	4,47852	5,71403	6,13205
C3 I3 O4	2,63010	4,69331	5,97946	6,40441
A2 I2 O3	2,49880	4,48051	5,71551	6,13181
A2 I2 O4	2,51972	4,51264	5,77352	6,18773
A3 I3 O3	2,53795	4,55658	5,83113	6,28436
	X=,2,Y=,4	X=,4,Y=,4	X=,6,Y=,4	X=,8,Y=,4
ACTUAL	1,44201	2,55533	3,23327	3,45907
C1 I3 O3+2	1,45995	2,58452	3,26775	3,49509
C1 I3 O4	1,54298	2,76094	3,51773	3,73985
C2 I2 O3	1,49652	2,64821	3,34715	3,57962
C2 I2 O3+2	1,46033	2,58508	3,26835	3,49568
C2 I2 O4	1,46752	2,61264	3,31441	3,54462
C3 I3 O3	1,47253	2,60928	3,30135	3,53887
C3 I3 O3+2	1,45932	2,58388	3,26786	3,49613
C3 I3 O4	1,53391	2,70115	3,42200	3,64793
A2 I2 O3	1,46222	2,58773	3,27090	3,49813
A2 I2 O4	1,47050	2,60448	3,31149	3,54465
A3 I3 O3	1,46563	2,63208	3,36436	3,65351
	X=,2,Y=,2	X=,4,Y=,2	X=,6,Y=,2	X=,8,Y=,2
ACTUAL	,49121	,85099	1,06225	1,13144
C1 I3 O3+2	,49892	,86055	1,07171	1,14075
C1 I3 O4	,52264	,91317	1,15346	1,23591
C2 I2 O3	,51031	,87890	1,09358	1,16376
C2 I2 O3+2	,49943	,86102	1,07207	1,14108
C2 I2 O4	,49858	,85574	1,07585	1,20181
C3 I3 O3	,50400	,86767	1,07603	1,14757
C3 I3 O3+2	,49866	,86073	1,07236	1,14142
C3 I3 O4	,51658	,88193	1,09550	1,16566
A2 I2 O3	,50184	,86269	1,07334	1,14224
A2 I2 O4	,50342	,85886	1,06018	1,21001
A3 I3 O3	,48820	,85894	1,13372	1,26152

TABLE 2.2

SQUARE PLATE BENDING PROBLEM, 5X5 GRID

ABSOLUTE ERRORS ((ACTUAL - CALCULATED) * 10**3)

	X=.2,Y=.8	X=.4,Y=.8	X=.6,Y=.8	X=.8,Y=.8
C1 I3 03+2	-.04558	-.09585	-.13308	-.14647
C1 I3 04	-.20685	-.40649	-.52622	-.62676
C2 I2 03	-.16621	-.31534	-.41540	-.45009
C2 I2 03+2	-.04594	-.09639	-.13363	-.14702
C2 I2 04	-.10214	-.18782	-.27020	-.28561
C3 I3 03	-.08220	-.18316	-.24660	-.24753
C3 I3 03+2	-.04771	-.10233	-.14135	-.15197
C3 I3 04	-.22347	-.39844	-.48763	-.55384
A2 I2 03	-.04789	-.09936	-.13678	-.15010
A2 I2 04	-.07250	-.12562	-.16561	-.19897
A3 I3 03	-.12722	-.20422	-.28286	-.27587

	X=.2,Y=.6	X=.4,Y=.6	X=.6,Y=.6	X=.8,Y=.6
C1 I3 03+2	-.02997	-.05559	-.07277	-.07877
C1 I3 04	-.16639	-.32246	-.45569	-.47566
C2 I2 03	-.10154	-.18530	-.23910	-.25745
C2 I2 03+2	-.03032	-.05615	-.07340	-.07942
C2 I2 04	-.06997	-.12834	-.16158	-.17677
C3 I3 03	-.05393	-.10707	-.14333	-.15947
C3 I3 03+2	-.03041	-.05644	-.07457	-.08244
C3 I3 04	-.16293	-.27123	-.34000	-.35480
A2 I2 03	-.03163	-.05843	-.07605	-.08220
A2 I2 04	-.05255	-.09056	-.13406	-.13812
A3 I3 03	-.07078	-.13450	-.19167	-.23475

	X=.2,Y=.4	X=.4,Y=.4	X=.6,Y=.4	X=.8,Y=.4
C1 I3 03+2	-.01794	-.02919	-.03448	-.03602
C1 I3 04	-.10097	-.20561	-.28446	-.28078
C2 I2 03	-.05450	-.09288	-.11388	-.12055
C2 I2 03+2	-.01832	-.02975	-.03508	-.03661
C2 I2 04	-.02551	-.05731	-.08114	-.08555
C3 I3 03	-.03052	-.05395	-.06808	-.07980
C3 I3 03+2	-.01731	-.02855	-.03459	-.03706
C3 I3 04	-.09190	-.14582	-.18873	-.18886
A2 I2 03	-.02021	-.03240	-.03763	-.03906
A2 I2 04	-.02849	-.04915	-.07822	-.08558
A3 I3 03	-.02361	-.07675	-.13109	-.19444

	X=.2,Y=.2	X=.4,Y=.2	X=.6,Y=.2	X=.8,Y=.2
C1 I3 03+2	-.00771	-.00956	-.00946	-.00931
C1 I3 04	-.03143	-.06218	-.09121	-.10447
C2 I2 03	-.01910	-.02791	-.03133	-.03232
C2 I2 03+2	-.00822	-.01003	-.00982	-.00964
C2 I2 04	-.00737	-.00475	-.01360	-.07037
C3 I3 03	-.01279	-.01668	-.01378	-.01613
C3 I3 03+2	-.00745	-.00974	-.01011	-.00998
C3 I3 04	-.02537	-.03094	-.03325	-.03422
A2 I2 03	-.01063	-.01170	-.01109	-.01080
A2 I2 04	-.01221	-.00787	.00207	-.07657
A3 I3 03	.00301	-.00795	-.07147	-.13008

TABLE 2,3
SQUARE PLATE BENDING PROBLEM, 5X5 GRID
PERCENT RELATIVE ERROR IN THE CALCULATED VALUES

	X=,2,Y=,8	X=,4,Y=,8	X=,6,Y=,8	X=,8,Y=,8
C1 I3 O3+2	-1,298	-1,517	-1,646	-1,687
C1 I3 O4	-5,891	-6,432	-6,510	-7,220
C2 I2 O3	-4,733	-4,990	-5,139	-5,185
C2 I2 O3+2	-1,308	-1,525	-1,653	-1,694
C2 I2 O4	-2,909	-2,972	-3,342	-3,290
C3 I3 O3	-2,341	-2,898	-3,051	-2,851
C3 I3 O3+2	-1,359	-1,619	-1,749	-1,751
C3 I3 O4	-6,364	-6,305	-6,032	-6,380
A2 I2 O3	-1,364	-1,572	-1,692	-1,729
A2 I2 O4	-2,065	-1,988	-2,049	-2,292
A3 I3 O3	-3,623	-3,232	-3,499	-3,178
	X=,2,Y=,6	X=,4,Y=,6	X=,6,Y=,6	X=,8,Y=,6
C1 I3 O3+2	-1,215	-1,257	-1,290	-1,302
C1 I3 O4	-6,744	-7,292	-8,080	-7,863
C2 I2 O3	-4,116	-4,190	-4,240	-4,256
C2 I2 O3+2	-1,229	-1,270	-1,302	-1,313
C2 I2 O4	-2,836	-2,902	-2,865	-2,922
C3 I3 O3	-2,186	-2,421	-2,542	-2,636
C3 I3 O3+2	-1,233	-1,276	-1,322	-1,363
C3 I3 O4	-6,604	-6,134	-6,029	-5,865
A2 I2 O3	-1,282	-1,321	-1,349	-1,359
A2 I2 O4	-2,130	-2,048	-2,377	-2,283
A3 I3 O3	-2,869	-3,042	-3,399	-3,880
	X=,2,Y=,4	X=,4,Y=,4	X=,6,Y=,4	X=,8,Y=,4
C1 I3 O3+2	-1,244	-1,142	-1,066	-1,041
C1 I3 O4	-7,002	-8,046	-8,798	-8,117
C2 I2 O3	-3,780	-3,635	-3,522	-3,485
C2 I2 O3+2	-1,270	-1,164	-1,085	-1,058
C2 I2 O4	-1,769	-2,243	-2,510	-2,473
C3 I3 O3	-2,116	-2,111	-2,106	-2,307
C3 I3 O3+2	-1,200	-1,117	-1,070	-1,072
C3 I3 O4	-6,373	-5,706	-5,837	-5,460
A2 I2 O3	-1,401	-1,268	-1,164	-1,129
A2 I2 O4	-1,975	-1,923	-2,419	-2,474
A3 I3 O3	-1,638	-3,003	-4,054	-5,621
	X=,2,Y=,2	X=,4,Y=,2	X=,6,Y=,2	X=,8,Y=,2
C1 I3 O3+2	-1,570	-1,123	-,891	-,823
C1 I3 O4	-6,399	-7,307	-8,587	-9,234
C2 I2 O3	-3,889	-3,280	-2,949	-2,857
C2 I2 O3+2	-1,674	-1,179	-,925	-,852
C2 I2 O4	-1,501	-,558	-1,280	-6,220
C3 I3 O3	-2,604	-1,960	-1,297	-1,426
C3 I3 O3+2	-1,517	-1,144	-,952	-,882
C3 I3 O4	-5,165	-3,636	-3,130	-3,025
A2 I2 O3	-2,164	-1,375	-1,044	-,955
A2 I2 O4	-2,486	-,925	,195	-6,945
A3 I3 O3	,612	-,934	-6,728	-11,497

TABLE 2.4

SQUARE PLATE BENDING PROBLEM, 5X5 GRID

METHODS SORTED BY MEAN RELATIVE ERROR (0/0)

1	C1 I3 O3+2	1,475
2	C2 I2 O3+2	1,484
3	A2 I2 O3	1,530
4	C3 I3 O3+2	1,541
5	A2 I2 O4	2,225
6	C3 I3 O3	2,724
7	C2 I2 O4	3,066
8	A3 I3 O3	3,582
9	C2 I2 O3	4,708
10	C3 I3 O4	6,109
11	C1 I3 O4	7,229

The remaining results all involved more than the 12 freedoms discussed above and all gave noticeably poorer results. In general, it does not appear that significant improvement is achieved by averaging polynomials.

2.3 Membrane Problem

The cantilever plate illustrated in Fig. 2.3 was analyzed in order to obtain purely membrane results. Both 3 by 5 and 5 by 9 uniform grids were used to obtain the results given in Table 2.5. Being a lower order equation

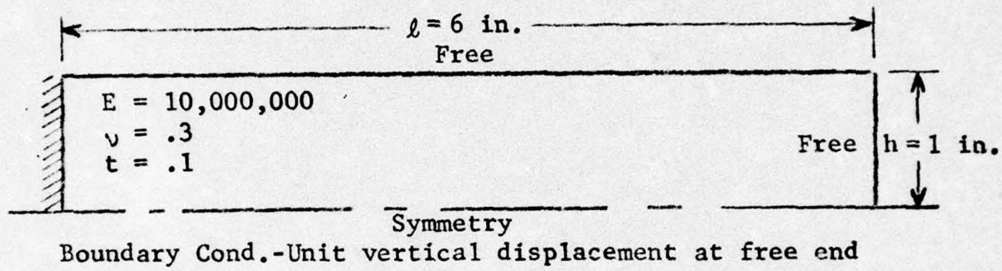


Fig. 2.3 Cantilever Plate

system, only quadratic (O 2) interpolation was used in this test. We note that the averaged polynomial results A 2 I 2 O 2 for the 3 x 5 were better than the non-averaged. This result appears to occur in rectangular grids when the neighbor patterns are not symmetric about the EIA. Figure 2.4 shows the 6-point neighbor pattern used for a typical control point of this problem.

Theoretically, the total applied force required to produce a uniform unit displacement at the free end is given by

$$P \ell \left((2 \ell / h)^2 + 2 (1 + \nu) \right) = E h t,$$

which yields $P = 8636$ for this problem. Thus, we notice from Table 2.5 that C 2 I 2 O 2 converges from the flexible side and A 2 I 2 O 2 from the stiff side. In the remainder of this report, we shall see that the cases C i I j O k are the more interesting and useful.

TABLE 2.5

Displacement Results along Symmetry Line of Cantilever Plate Problem and Total Applied Force

3 x 5 Grid		5 x 9 Grid	
C2 I2 02	A2 I2 02	C2 I2 01	A2 I2 02
.0758	.0871	.0900	.0920
.3065	.3159	.3227	.3211
.6488	.6357	.6396	.6384
1.0	1.0	1.0	1.0
8350	9008	8447	8738

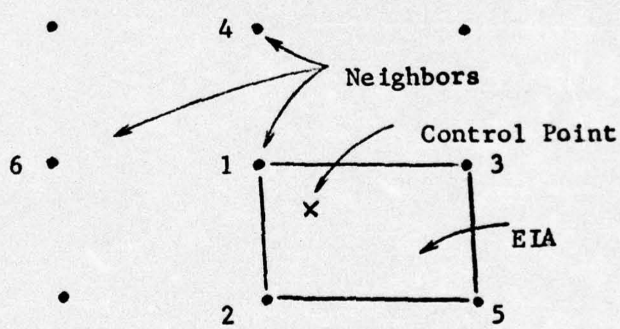


Fig. 2.4 Typical 6-Point Neighbor Pattern

Section 3

DISC PROBLEM

The disc problem provides an appropriate case for testing a more complex grid structure without the added complexity of coupled bending and membrane effects. In this section we provide results for disc bending tests using two grids. The conclusions are slightly different from those for the uniform rectangular grid of the previous section.

3.1 Problem Definition

The specific problem used is illustrated in Fig. 3.1. The analytic solution for the normal displacement in this problem is given by

$$w(x, y) = c\,(R^2 - x^2 - y^2)^2 ,$$

where

$$c = 3q\,(1 - \nu^2)/(16\,E\,h^3)$$
$$= 6.25 \times 10^{-4}$$

and

$$R = 2 .$$

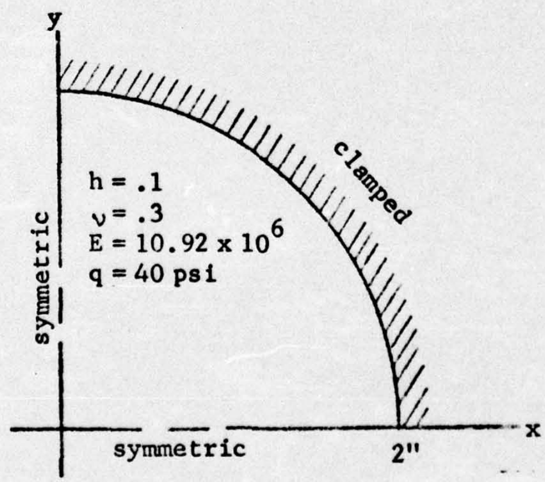


Fig. 3.1 Disc Problem

Two grids, designated G and B for "good" and "bad" were used for the numerical solution to this problem. Grid B, Fig. 3.3, is obtained from a square grid by the expression

$$\underset{\sim}{p}' = s\frac{\|\underset{\sim}{p}\|_\infty}{\|\underset{\sim}{p}\|_2}\cdot\underset{\sim}{p} ,$$

where $\underset{\sim}{p} = (x, y)$ represents any point on the square grid, s is a scale factor, and $\underset{\sim}{p}' = (x', y')$ represents the corresponding point on the disc. The norms are given by

$$\|\underset{\sim}{p}\|_\infty = \max (|x| , |y|) ,$$

$$\|\underset{\sim}{p}\|_2 = \sqrt{x^2 + y^2} .$$

Grid G, Fig. 3.2, was obtained from B by adjusting the internal diagonal points so that the quadrilateral adjacent to the origin is square.

Bending results were obtained for this problem using varying numbers of control points and integration points in each EIA as described in Section 2.1. These results appear in Tables 3.1 - 3.8, using the notation introduced in Section 2.1 to identify the cases.

3.2 Observations on Disc Problem

From the negative signs of the errors in Tables 3.2 and 3.6 we note that the finite difference model is more flexible than the physical model. This is consistent with the results for the rectangular plate discussed in Section 2.2.

Of course, we should be pleased if one modus operandi for this analysis system could be shown to perform well for any grid. Unfortunately, we have not found a theoretical basis for establishing one and so we must look to experience. If we let the mean relative error be our guide, we obtain suggestive results by comparing Tables 2.4, 3.4, and 3.8.

The best overall performance was obtained from C 2 I 2 0 3+2 . For a reasonably good grid, C 1 I 3 0 3+2 also performed very well but suffered

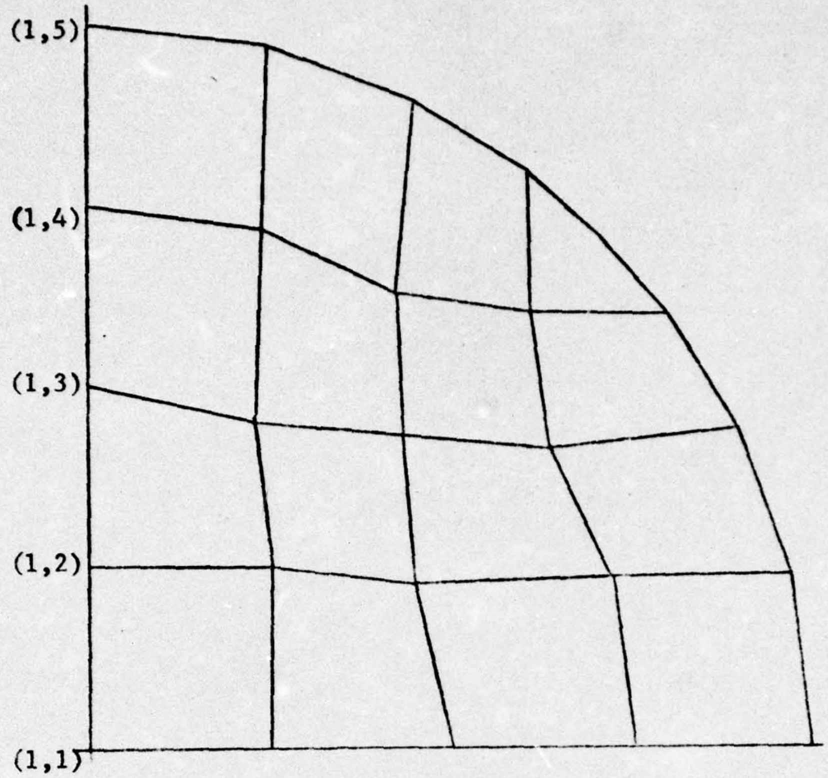


Fig. 3.2 Grid G for the Disc Problem

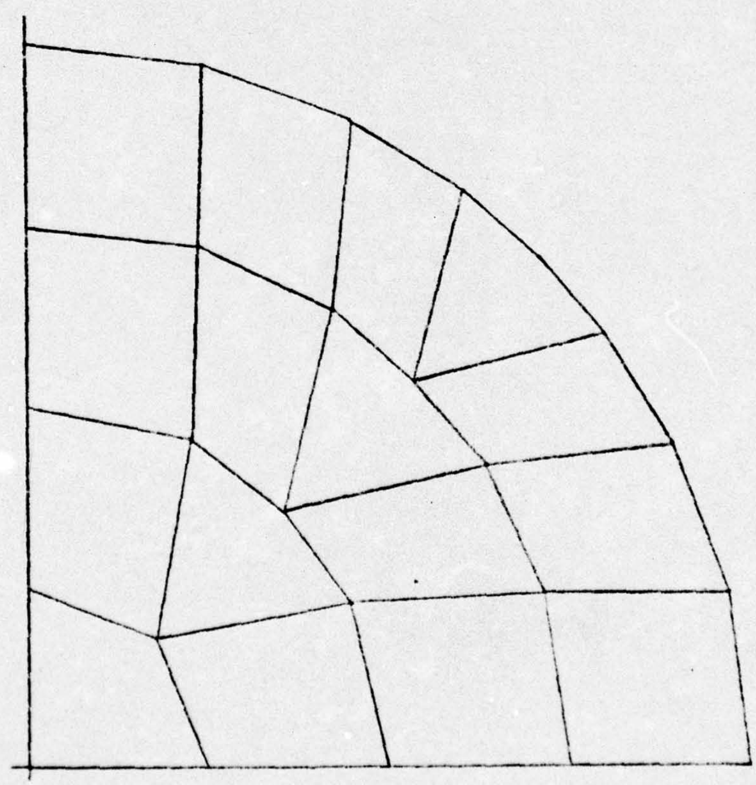


Fig. 3.3 Grid B for the Disc Problem. Nearly triangular EIA's along the diagonal.

TABLE 3,1

CLAMPED DISC BENDING PROBLEM, 5X5 GRID G

CALCULATED VALUES (* 10**2)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
ACTUAL	,19141	,19141	,19141	,07368
C1 I3 03+2	,21512	,21106	,19926	,06647
C2 I2 03+2	,21334	,20415	,20615	,07402
C2 I2 03	,20557	,20111	,20632	,06926
C3 I3 03	,20227	,19713	,19613	,07184
A2 I2 03	,26712	,26230	,23385	,08323
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
ACTUAL	,56250	,56250	,40414	,19141
C1 I3 03+2	,60445	,59954	,42652	,19926
C2 I2 03+2	,59624	,60201	,42186	,20371
C2 I2 03	,59652	,60097	,42078	,20639
C3 I3 03	,58986	,58693	,42165	,19661
A2 I2 03	,73925	,75013	,51613	,23419
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
ACTUAL	,87891	,76562	,56250	,19141
C1 I3 03+2	,92977	,81026	,59954	,21106
C2 I2 03+2	,92566	,80301	,60229	,20388
C2 I2 03	,92014	,80606	,59960	,20114
C3 I3 03	,91741	,78927	,58826	,19752
A2 I2 03	1,12564	,98672	,77992	,26456
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
ACTUAL	1,00000	,87891	,56250	,19141
C1 I3 03+2	1,05346	,92977	,60445	,21512
C2 I2 03+2	1,04612	,92683	,60056	,21157
C2 I2 03	1,04275	,93094	,59810	,20494
C3 I3 03	1,02752	,91628	,59014	,20259
A2 I2 03	1,26945	,11967	,77636	,27198

TABLE 3,2
CLAMPED DISC BENDING PROBLEM, 5X5 GRID G
ABSOLUTE ERRORS ((ACTUAL - CALCULATED) * 10**2)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 O3+2	-,02371	-,01965	-,00785	,00721
C2 I2 O3+2	-,02193	-,01274	-,01474	-,00034
C2 I2 O3	-,01416	-,00970	-,01491	,00442
C3 I3 O3	-,01066	-,00572	-,00472	,00184
A2 I2 O3	-,07571	-,07089	-,04244	-,00955

	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 O3+2	-,04195	-,03704	-,02238	-,00785
C2 I2 O3+2	-,03374	-,03951	-,01772	-,01230
C2 I2 O3	-,03402	-,03847	-,01664	-,01498
C3 I3 O3	-,02736	-,02443	-,01751	-,00520
A2 I2 O3	-,17675	-,18763	-,11199	-,04278

	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 O3+2	-,05086	-,04464	-,03704	-,01965
C2 I2 O3+2	-,04675	-,03739	-,03979	-,01247
C2 I2 O3	-,04123	-,04044	-,03710	-,00973
C3 I3 O3	-,03850	-,02365	-,02576	-,00611
A2 I2 O3	-,24673	-,22110	-,21742	-,07315

	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 O3+2	-,05346	-,05086	-,04195	-,02371
C2 I2 O3+2	-,04612	-,04792	-,03806	-,02016
C2 I2 O3	-,04275	-,05203	-,03560	-,01353
C3 I3 O3	-,02752	-,03737	-,02764	-,01118
A2 I2 O3	-,26945	,75924	-,21386	-,08057

TABLE 3,3
CLAMPED DISC BENDING PROBLEM, 5X5 GRID G
PERCENT RELATIVE ERROR IN THE CALCULATED VALUES

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 O3+2	-12,387	-10,266	-4,101	9,789
C2 I2 O3+2	-11,457	-6,656	-7,701	-,457
C2 I2 O3	-7,398	-5,068	-7,790	6,003
C3 I3 O3	-5,674	-2,988	-2,466	2,501
A2 I2 O3	-39,554	-37,036	-22,172	-12,957
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 O3+2	-7,458	-6,585	-5,538	-4,101
C2 I2 O3+2	-5,998	-7,024	-4,385	-6,426
C2 I2 O3	-6,048	-6,839	-4,117	-7,826
C3 I3 O3	-4,864	-4,343	-4,333	-2,717
A2 I2 O3	-31,422	-33,356	-27,711	-22,350
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 O3+2	-5,787	-5,831	-6,585	-10,266
C2 I2 O3+2	-5,319	-4,884	-7,074	-6,515
C2 I2 O3	-4,691	-5,282	-6,596	-5,083
C3 I3 O3	-4,380	-3,089	-4,580	-3,192
A2 I2 O3	-28,072	-28,879	-38,652	-38,216
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 O3+2	-5,346	-5,787	-7,458	-12,387
C2 I2 O3+2	-4,612	-5,452	-6,766	-10,532
C2 I2 O3	-4,275	-5,920	-6,329	-7,069
C3 I3 O3	-2,752	-4,252	-4,914	-5,841
A2 I2 O3	-26,945	86,385	-38,020	-42,093

TABLE 3,4
CLAMPED DISC BENDING PROBLEM, 5X5 GRID G
METHODS SORTED BY MEAN RELATIVE ERROR (0/0)

1	C3 I3 O3	3,996
2	C2 I2 O3	5,498
3	C2 I2 O3+2	5,714
4	C1 I3 O3+2	6,278
5	A2 I2 O3	44,723

TABLE 3,5
CLAMPED DISC BENDING PROBLEM, 5X5 GRID B
CALCULATED VALUES (* 10**2)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
ACTUAL	,19141	,19141	,19141	,19141
C1 I3 O3+2	,22876	,22875	,21732	,20792
C2 I2 O3	,20871	,20048	,19994	,19921
C2 I2 O3+2	,21405	,20425	,20629	,19883
C3 I3 O3	,20545	,20144	,20287	,20261
A2 I2 O3	,25946	,26263	,22873	,21216

	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
ACTUAL	,56250	,56250	,56250	,19141
C1 I3 O3+2	,66104	,66221	,65773	,21732
C2 I2 O3	,60882	,59936	,59647	,19933
C2 I2 O3+2	,61798	,61243	,60826	,20714
C3 I3 O3	,61531	,61058	,60582	,20218
A2 I2 O3	,74320	,68303	,67106	,22336

	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
ACTUAL	,87891	,87891	,56250	,19141
C1 I3 O3+2	1,03387	1,03991	,66221	,22875
C2 I2 O3	,93519	,94318	,59708	,19957
C2 I2 O3+2	,94784	,96929	,61204	,20394
C3 I3 O3	,97307	,98106	,61195	,20275
A2 I2 O3	1,05857	1,06215	,68300	,26451

	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
ACTUAL	1,00000	,87891	,56250	,19141
C1 I3 O3+2	1,17421	1,03387	,66104	,22876
C2 I2 O3	1,07442	,92728	,60618	,20699
C2 I2 O3+2	1,09174	,94751	,61689	,21486
C3 I3 O3	1,10015	,97354	,61423	,20350
A2 I2 O3	1,28576	1,04681	,74856	,25730

TABLE 3,6

CLAMPED DISC BENDING PROBLEM, 5X5 GRID B

ABSOLUTE ERRORS ((ACTUAL - CALCULATED) * 10**2)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 03+2	-,03735	-,03734	-,02591	-,01651
C2 I2 03	-,01730	-,00907	-,00853	-,00780
C2 I2 03+2	-,02264	-,01284	-,01488	-,00742
C3 I3 03	-,01404	-,01003	-,01146	-,01120
A2 I2 03	-,06805	-,07122	-,03732	-,02075

	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 03+2	-,09854	-,09971	-,09523	-,02591
C2 I2 03	-,04632	-,03686	-,03397	-,00792
C2 I2 03+2	-,05548	-,04993	-,04576	-,01573
C3 I3 03	-,05281	-,04808	-,04332	-,01077
A2 I2 03	-,18070	-,12053	-,10856	-,03195

	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 03+2	-,15496	-,16100	-,09971	-,03734
C2 I2 03	-,05628	-,06427	-,03458	-,00816
C2 I2 03+2	-,06893	-,09038	-,04954	-,01253
C3 I3 03	-,09416	-,10215	-,04945	-,01134
A2 I2 03	-,17966	-,18324	-,12050	-,07310

	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 03+2	-,17421	-,15496	-,09854	-,03735
C2 I2 03	-,07442	-,04837	-,04368	-,01558
C2 I2 03+2	-,09174	-,06860	-,05439	-,02345
C3 I3 03	-,10015	-,09463	-,05173	-,01209
A2 I2 03	-,28576	-,16790	-,18606	-,06589

TABLE 3,7
CLAMPED DISC BENDING PROBLEM, 5X5 GRID B
PERCENT RELATIVE ERROR IN THE CALCULATED VALUES

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 O3+2	-19,513	-19,508	-13,536	-8,625
C2 I2 O3	-9,038	-4,739	-4,456	-4,075
C2 I2 O3+2	-11,828	-6,710	-7,777	-3,876
C3 I3 O3	-7,335	-5,240	-5,987	-5,851
A2 I2 O3	-35,552	-37,208	-19,497	-10,841
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 O3+2	-17,518	-17,726	-16,930	-13,536
C2 I2 O3	-8,235	-6,553	-6,039	-4,138
C2 I2 O3+2	-9,863	-8,876	-8,135	-6,218
C3 I3 O3	-9,388	-8,548	-7,701	-5,627
A2 I2 O3	-32,124	-21,428	-19,300	-10.692
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 O3+2	-17,631	-18,318	-17,726	-19.508
C2 I2 O3	-6,403	-7,312	-6.148	-4.263
C2 I2 O3+2	-7,843	-10,283	-8,807	-6,546
C3 I3 O3	-10,713	-11,622	-8,791	-5,924
A2 I2 O3	-20,441	-20,849	-21.422	-38,190
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 O3+2	-17,421	-17,631	-17,518	-19,513
C2 I2 O3	-7,442	-5,503	-7,766	-8,140
C2 I2 O3+2	-9,174	-7,805	-9,669	-12,251
C3 I3 O3	-10,015	-10,767	-9,196	-6,316
A2 I2 O3	-28,576	-19,103	-33,077	-34,423

TABLE 3,8
CLAMPED DISC BENDING PROBLEM, 5X5 GRID B
METHODS SORTED BY MEAN RELATIVE ERROR (0/0)

1	C2 I2 O3	6,798
2	C2 I2 O3+2	8,920
3	C3 I3 O3	9,978
4	C1 I3 O3+2	17,608
5	A2 I2 O3	24,363

greatly (Table 3.8) for the poor grid. Probably the ideal form would be C2 I3 O3+2, however, due to complexities in the implementation it has not yet been developed. Very briefly, the complexity involved is the suitable correlation of the 2 x 2 array of control points with the 3 x 3 array of integration points on each EIA for numerical integration. This is a development to be made to STAGS.

Section 4
SPHERICAL CAP PROBLEM

The most difficult test of this series involves a shallow spherical cap requiring consideration of both bending and membrane effects. In this section we analyze both an axisymmetric cap problem and one that is not. In the first case we use a converged solution obtained by the shell of revolution program BOSOR[2] as the "actual" solution for purposes of error calculation. In the second case, we have no "actual" solution and simply present a number of results from the STAGS-FIDAG system, a new version of STAGS and a finite element program REXBAT[8].

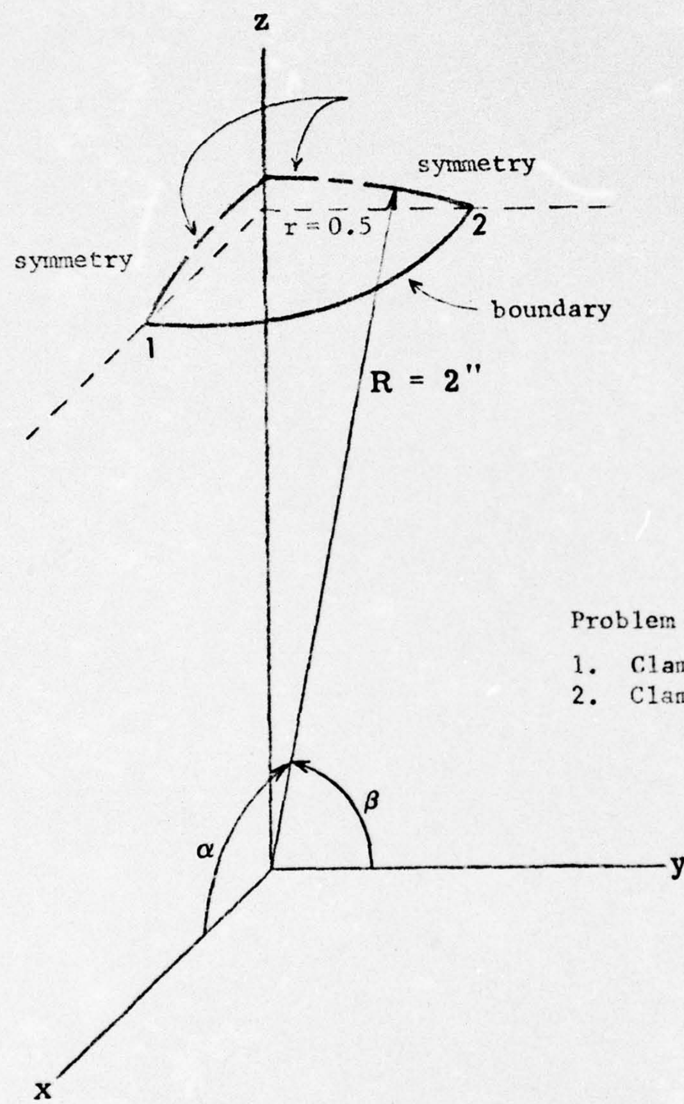
4.1 Problem Definition

The basic problem is treated as a quarter cap with symmetry conditions along the cuts as illustrated in Figure 4.1.

The grid for this problem was obtained by simply projecting the grid used for the disc up to the cap, using spherical coordinates α and β. The α, β coordinates of any vector defining a point on the sphere are the angles between the vector and the x-axis and y-axis as illustrated in Figure 4.1.

We recognize that this grid is not ideal for either problem. Indeed, since problem 1 is axisymmetric, one needs only a one-dimensional grid. Similarly, for problem 2 one needs to have grid points concentrated around the point supports in order to obtain an accurate representation there. For problem 1 we did use the idealized grid with the BOSOR program to obtain an accurate solution. The two-dimensional grid then provided a means for conveniently testing the behavior of the STAGS-FIDAG program on a combined bending-membrane problem.

Similarly, for problem 2 we were able to conveniently create a test environment for the program which, even though it was not ideal for the problem,



Problem

1. Clamped along boundary.
2. Clamped at points 1 and 2 only.

E	= 10,000,000	Young's modulus
ν	= .3	Poisson's ratio
q	= 1 psi	Uniform normal pressure
h	= .05 in.	Thickness

Fig. 4.1 Spherical Cap Problem

provided results indicative of the program operation. Figure 4.2 shows the appearance of the grid on the cap looking down at the pole. The quadrilaterals used in the STAGS-FIDAG analysis have corner points $(i, i+1, i+n+1, i+n)$, where n is the number of points in the radial direction and $i = (j-1)n+k$ for any $j, k = 1, 2, \ldots, n-1$. The diagonal lines in Figure 4.2 on each EIA were introduced by REXBAT for calculating force distribution as discussed below.

4.2 Comparative Finite Element Analysis

A conforming quadrilateral element [3] was used by the REXBAT finite element analysis program for this problem. Although this element is rather expensive to use, it provides very accurate results which are particularly useful for the evaluation of the STAGS-FIDAG system. At the present time, the REXBAT implementation of this quadrilateral element does not have a consistent force distribution capability and consequently, monotonic convergence from the "stiff" side cannot be expected to strictly hold. The force distribution is obtained in REXBAT by dividing each quadrilateral into two triangles as shown in Figure 4.2, and using a consistent triangular element distribution.

4.3 Numerical Results

The results of the STAGS-FIDAG and REXBAT tests appear in Tables 4.1 - 4.11. In the tables, the point $(x, y) = (1, 1)$ corresponds to the pole, node 1 in Figure 4.2. In general, a point (x, y) in a table corresponds to node

$$1 + (x + n \cdot y - 1 - n)(n-1)/4$$

in the corresponding grid having n radial nodes. The grid in Figure 4.2 has $n = 9$.

As long as the number $n-1$ of radial EIA's is a multiple of 4, the grid has non-diagonal points which coincide with those of the 5x5 grid and, consequently, those entries in the tables correspond. Recall, however, that the grid generator adjusts the diagonal nodes corresponding to $x = y$, $x = 2, \ldots,$ for the grid improvement and so the diagonal entries in the tables <u>do not</u> correspond for different grids.

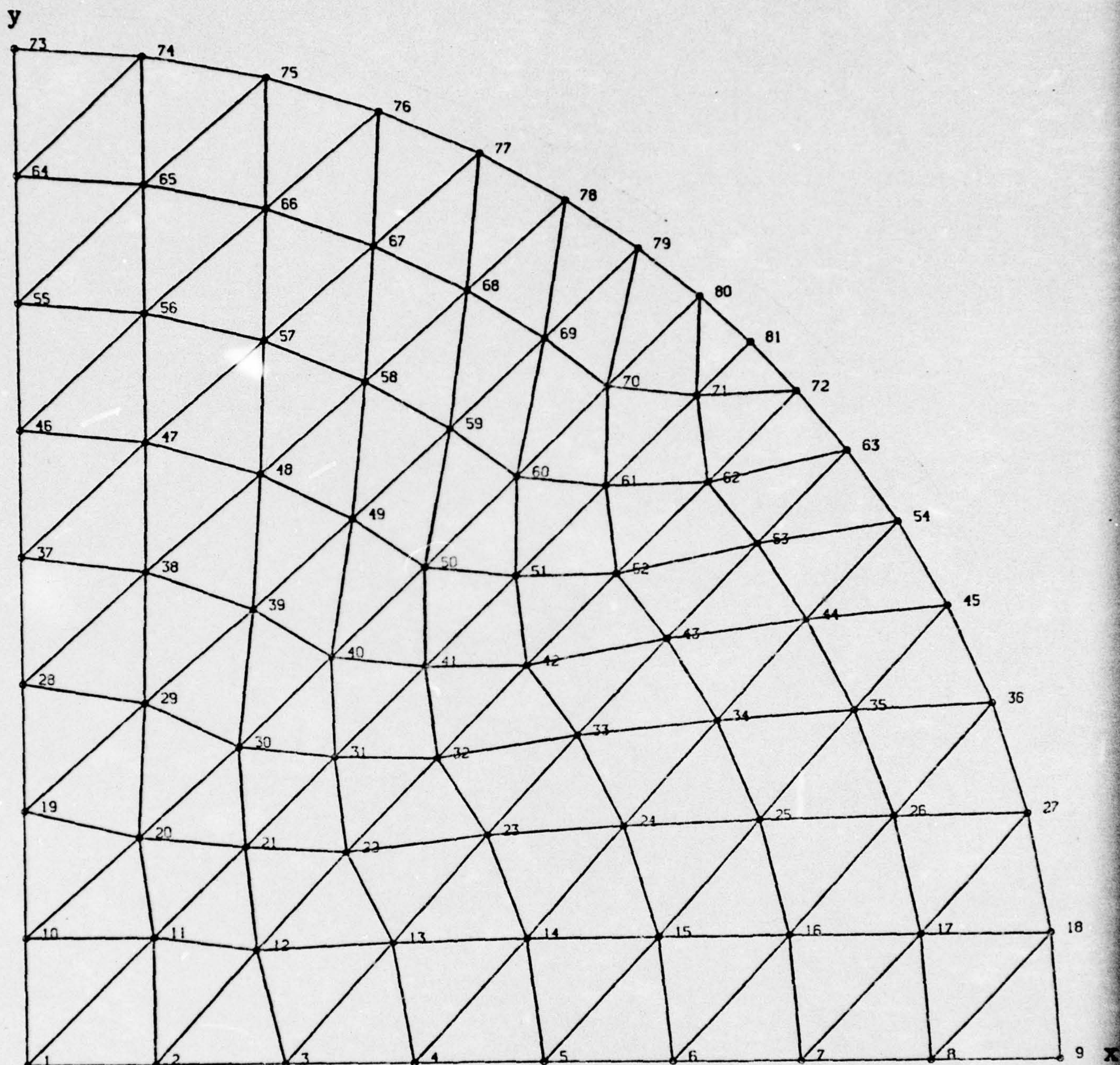


Fig. 4.2 9 x 9 Cap Grid Projection in Cartesian Coordinates

TABLE 4.1

CLAMPED SPHERICAL CAP PROBLEM, 5X5 GRID

CALCULATED VALUES (* 10**6)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
ACTUAL	,76580	,76580	,76580	,31480
C1 I3 03+2	,85240	,83380	,78621	,27400
C2 I2 03+2	,85222	,79738	,80061	,29921
REXBAT Q	,76148	,76715	,77714	,30380
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
ACTUAL	2,14900	2,14900	1,59700	,76580
C1 I3 03+2	2,25600	2,23770	1,62710	,78560
C2 I2 03+2	2,24820	2,25680	1,59890	,80326
REXBAT Q	2,18990	2,20180	1,61620	,77714
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
ACTUAL	3,25500	2,87900	2,14900	,76580
C1 I3 03+2	3,33700	2,94810	2,23760	,83510
C2 I2 03+2	3,39680	2,96760	2,25950	,80413
REXBAT Q	3,34220	2,94420	2,20180	,76715
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
ACTUAL	3,66300	3,25500	2,14900	,76580
C1 I3 03+2	3,73210	3,33700	2,25760	,85570
C2 I2 03+2	3,81660	3,40120	2,25750	,83248
REXBAT Q	3,78460	3,34220	2,18990	,76148

TABLE 4.2

CLAMPED SPHERICAL CAP PROBLEM, 5X5 GRID

PERCENT RELATIVE ERROR IN THE CALCULATED VALUES

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 03+2	-11.308	-8.880	-2.665	12.961
C2 I2 03+2	-11.285	-4.124	-4.546	4.952
REXBAT Q	.564	-.176	-1.481	3.494
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 03+2	-4.979	-4.128	-1.885	-2.586
C2 I2 03+2	-4.616	-5.016	-.119	-4.892
REXBAT Q	-1.903	-2.457	-1.202	-1.481
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 03+2	-2.519	-2.400	-4.123	-9.049
C2 I2 03+2	-4.356	-3.077	-5.142	-5.005
REXBAT Q	-2.679	-2.265	-2.457	-.176
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 03+2	-1.886	-2.519	-5.054	-11.739
C2 I2 03+2	-4.193	-4.492	-5.049	-8.707
REXBAT Q	-3.320	-2.679	-1.903	.564

TABLE 4.3

CLAMPED SPHERICAL CAP PROBLEM, 5X5 GRID

METHODS SORTED BY MEAN RELATIVE ERROR (070)

1	REXBAT Q	2.551
2	C1 I3 03+2	3.657
3	C2 I2 03+2	4.481

TABLE 4,4

CLAMPED SPHERICAL CAP PROBLEM, 9X9 GRID

CALCULATED VALUES (* 10**6)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
ACTUAL	,76580	,76580	,76580	,53300
C1 I3 03+2	,82200	,82910	,82410	,54570
C2 I2 03+2	,80640	,79620	,79490	,53470
REXBAT Q	,77863	,78052	,78343	,53096
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
ACTUAL	2,14900	2,14900	1,88700	,76580
C1 I3 03+2	2,25930	2,27860	1,97890	,82520
C2 I2 03+2	2,24250	2,23690	1,93670	,79920
REXBAT Q	2,17650	2,18080	1,89820	,78343
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
ACTUAL	3,25500	3,08600	2,14900	,76560
C1 I3 03+2	3,39750	3,21560	2,28220	,83310
C2 I2 03+2	3,37420	3,19840	2,25000	,79590
REXBAT Q	3,28840	3,11040	2,18080	,78052
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
ACTUAL	3,66300	3,25500	2,14900	,76580
C1 I3 03+2	3,79810	3,39940	2,26580	,82980
C2 I2 03+2	3,79960	3,38940	2,24590	,80610
REXBAT Q	3,69970	3,28840	2,17650	,77863

TABLE 4.5

CLAMPED SPHERICAL CAP PROBLEM, 9X9 GRID

PERCENT RELATIVE ERROR IN THE CALCULATED VALUES

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 03+2	-7,339	-8,266	-7,613	-2,383
C2 I2 03+2	-5,302	-3,970	-3,800	-,319
REXBAT Q	-1,675	-1,922	-2,302	,383
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 03+2	-5,133	-6,031	-4,870	-7,757
C2 I2 03+2	-4,351	-4,090	-2,634	-4,361
REXBAT Q	-1,280	-1,480	-,594	-2,302
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 03+2	-4,378	-4,200	-6,198	-8,788
C2 I2 03+2	-3,662	-3,642	-4,700	-3,931
REXBAT Q	-1,026	-,791	-1,480	-1,922
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 03+2	-3,688	-4,436	-5,435	-8,357
C2 I2 03+2	-3,729	-4,129	-4,509	-5,262
REXBAT Q	-1,002	-1,026	-1,280	-1,675

TABLE 4.6

CLAMPED SPHERICAL CAP PROBLEM, 9X9 GRID

METHODS SORTED BY MEAN RELATIVE ERROR (070)

1	REXBAT Q	1,148
2	C2 I2 03+2	3,950
3	C1 I3 03+2	4,898

TABLE 4.7
CLAMPED SPHERICAL CAP PROBLEM, 13X13 GRID
CALCULATED VALUES (* 10**6)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
ACTUAL	.76580	.76580	.76580	.61240
C1 I3 03+2	.81280	.82130	.82830	.63900
REXBAT Q	.78146	.78230	.78365	.61277
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
ACTUAL	2.14900	2.14900	1.98300	.76580
C1 I3 03+2	2.25650	2.28310	2.09640	.82880
REXBAT Q	2.17320	2.17520	1.98860	.78365
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
ACTUAL	3.25500	3.14700	2.14900	.76580
C1 I3 03+2	3.40690	3.30100	2.28340	.82270
REXBAT Q	3.27480	3.16010	2.17520	.78230
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
ACTUAL	3.66300	3.25500	2.14900	.76580
C1 I3 03+2	3.82020	3.40680	2.25650	.81610
REXBAT Q	3.68050	3.27480	2.17320	.78146

TABLE 4.8
CLAMPED SPHERICAL CAP PROBLEM, 13X13 GRID
PERCENT RELATIVE ERROR IN THE CALCULATED VALUES

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 03+2	-6.137	-7.247	-8.161	-4.344
REXBAT Q	-2.045	-2.155	-2.331	-.060
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 03+2	-5.002	-6.240	-5.719	-8.227
REXBAT Q	-1.126	-1.219	-.282	-2.331
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 03+2	-4.667	-4.894	-6.254	-7.430
REXBAT Q	-.608	-.416	-1.219	-2.155
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 03+2	-4.292	-4.664	-5.095	-6.568
REXBAT Q	-.478	-.608	-1.126	-2.045

TABLE 4.9
CLAMPED SPHERICAL CAP PROBLEM, 13X13 GRID
METHODS SORTED BY MEAN RELATIVE ERROR (070)

1	REXBAT Q	.882
2	C1 I3 03+2	5.134

TABLE 4,10
SPHERICAL CAP, 4 POINT SUPPORT, 9X9 GRID
CALCULATED VALUES (* 10**5)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 O3+2	,46750	,62720	,81900	,84920
C2 I2 O3+2	,48770	,65820	,85610	,89940
REXBAT Q	,39844	,55120	,72520	,76183
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 O3+2	,87480	,94090	,94970	,81670
C2 I2 O3+2	,90390	,96990	,97980	,85780
REXBAT Q	,77501	,83533	,84396	,72520
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 O3+2	1,08630	1,07090	,93910	,62340
C2 I2 O3+2	1,11570	1,10200	,97630	,66010
REXBAT Q	,97538	,96099	,83533	,55120
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 O3+2	1,14410	1,08550	,87220	,46460
C2 I2 O3+2	1,17790	1,12090	,90880	,49150
REXBAT Q	1,03380	,97538	,77501	,39844

TABLE 4,11
SPHERICAL CAP, 4 POINT SUPPORT, 13X13 GRID
CALCULATED VALUES (* 10**5)

	X= 1,Y= 4	X= 2,Y= 4	X= 3,Y= 4	X= 4,Y= 4
C1 I3 O3+2	,50960	,67370	,87460	,91460
REXBAT Q	,44870	,60452	,78298	,82201
	X= 1,Y= 3	X= 2,Y= 3	X= 3,Y= 3	X= 4,Y= 3
C1 I3 O3+2	,94120	1,01150	1,03184	,87300
REXBAT Q	,84707	,90764	,92276	,78298
	X= 1,Y= 2	X= 2,Y= 2	X= 3,Y= 2	X= 4,Y= 2
C1 I3 O3+2	1,16640	1,16020	1,01020	,67110
REXBAT Q	1,05580	1,04800	,90764	,60452
	X= 1,Y= 1	X= 2,Y= 1	X= 3,Y= 1	X= 4,Y= 1
C1 I3 O3+2	1,22930	1,16570	,93940	,50730
REXBAT Q	1,11640	1,05580	,84707	,44870

Based on experience with the plate and disc problems, methods C 1 I 3 0 3+2 and C 2 I 2 0 3+2 were used in this series of tests for the bending. We recognize that the general quartic integration (I 3) is beneficial and that more than one control point per EIA (C 2) is also beneficial. It appears that the ideal combination for accuracy would be C 2 I 3 0 3+2 which has not yet been implemented.

4.3.1 Clamped Cap Problem

The results for the clamped cap, problem 1, appear in Tables 4.1 - 4.9 along with "actual" results which were obtained from BOSOR as discussed earlier.

Notice in Tables 4.2, 4.5 and 4.8 that the REXBAT solutions do not converge from the "stiff" side, i.e., the coarse grids produce a solution which is greater than the actual at most points, as was anticipated from the discussion in Section 4.2. From these tables we also notice that the STAGS-FIDAG model is predominantly more flexible, which is consistent with the results of the plate and disc studies. We shall observe a reversal of this trend in the results for problem 2 below. We notice in Tables 4.6 and 4.9 that both STAGS-FIDAG results and the REXBAT results have essentially converged with the 9 x 9 grid to mean errors of 5% and 1%, respectively. Also, we observe from the similarity of the results for the two STAGS-FIDAG forms that both are reasonably good and neither is clearly superior. This is suggestive of a probable benefit from utilizing the best of both forms via C 2 I 2 0 3+2 as mentioned earlier.

One surprising observation is the occasional appearance of a slight increase in mean relative error in the STAGS-FIDAG results when the grid is refined. For example, compare the results for C 1 0 3 0 3+2 in Tables 4.3, 4.6 and 4.9. All we can conclude from these results is that convergence is not monotonic.

4.3.2 Point Support Problem

Our results for this problem are less detailed than previous results since the true solution is not known. In Tables 4.10 and 4.11 we note that all of the solutions increase as the grid is refined. This is traditional behavior for finite elements but is unusual for finite differences. It is the only case in this study for which the finite difference solutions converge from the stiff side.

Since a thorough convergence study was beyond the budgetary scope of this effort, further analysis was conducted with STAGS as reported in Section 4.4. In addition, crude projections based on REXBAT results from the 5 x 5, the 9 x 9 and the 13 x 13 grid were made. These appear in Figure 4.3. Both studies suggest that the 13 x 13 STAGS-FIDAG result is close to being correct.

4.4 STAGS Analysis

The grid described in Section 4.2 cannot be used in STAGS without FIDAG. The approach used for a spherical cap or disc with STAGS involves a "geographic" coordinate system of longitudes and latitudes. Since the longitudes coalesce at the pole, it is necessary to perturb the problem by introducing a small hole at the pole and applying suitable boundary conditions there. The best results obtained to date have been with free edge conditions around the hole.

The cap considered in this report corresponds to a maximum latitude of about 14.5°. For the STAGS study, the polar opening used was 1° latitude. The errors in the STAGS C′ solution for the clamped cap are given in Table 4.12 along one longitude. The STAGS C′ results for the four-point support cap along a longitude of a support are interpolated to the latitudes corresponding to the results of Table 4.11 and presented with those results in Table 4.13.

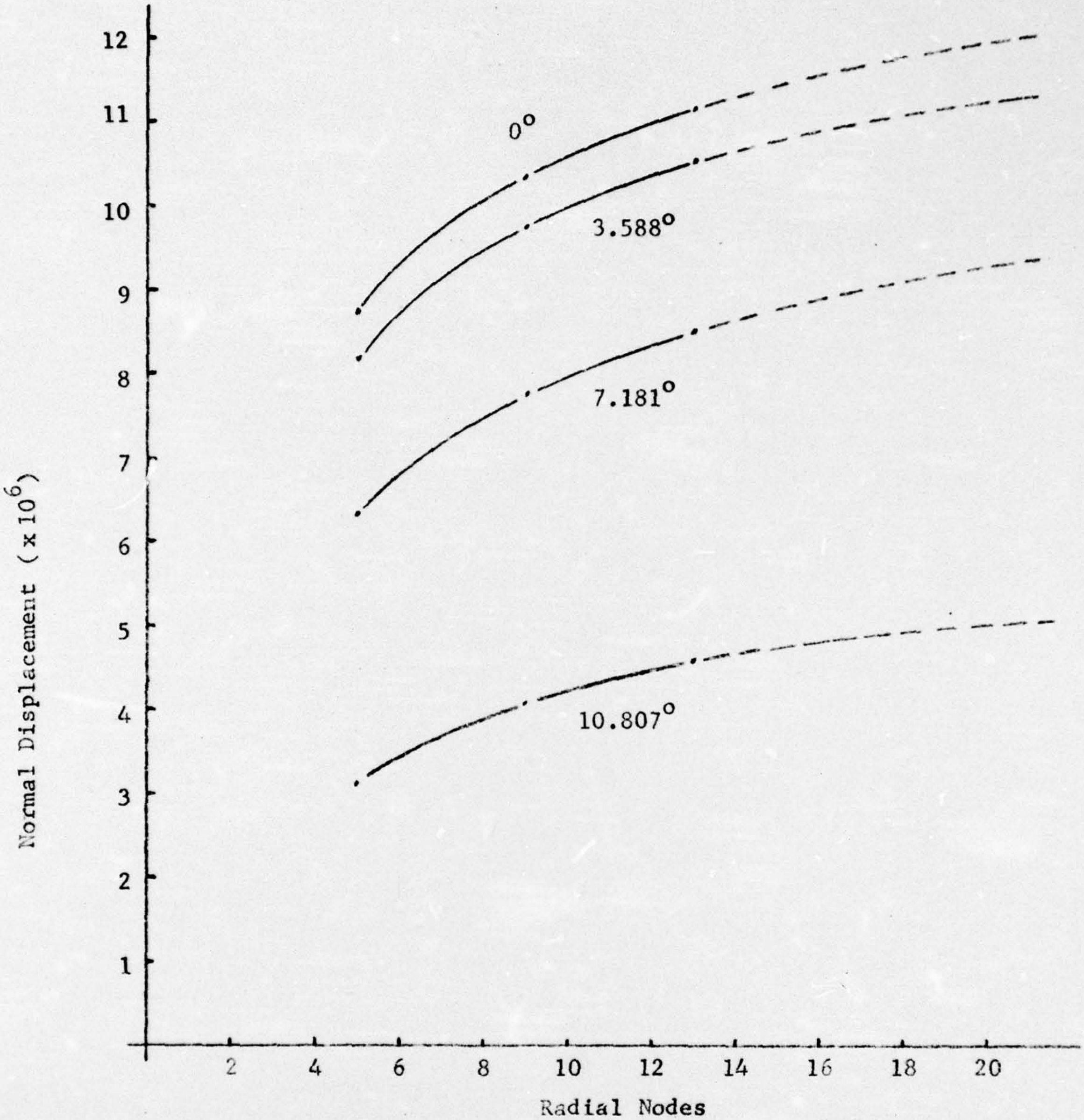


Fig. 4.3 Projected Solution from REXBAT Results for 4-Point Support Cap at Various Latitudes along a Longitude of Support

TABLE 4.12

Errors in the STAGS C′ Results

for the Clamped Cap at Various Latitudes (x 10^6)

No. Radial Points	Latitude (Degrees) 1.	4.375	7.75	11.125
Absolute				
5	-.067	.024	.007	-.0048
9	-.186	-.055	-.027	-.0129
17	-.192	0.054	0.022	0.0102
% Relative				
5	-1.84	.78	.08	-.71
9	-5.12	-1.80	-.31	-1.90
17	-5.28	-1.76	-.25	-1.50

TABLE 4.13

Comparison of Calculated Results

Along Supported Longitude of 4-Point Support Cap (x 10^5)

STAGS Grid Spec. - Radial x Circumferential

Case	Latitude (Degrees) 0	3.5883	7.1808	10.807
STAGS-FIDAG 13 x 13	1.229	1.166	.9403	.5084
STAGS C′ 13 x 25	1.243	1.142	.9221	.5077
STAGS C′ 13 x 13	1.118	1.024	.8543	.4332
REXBAT 13 x 13	1.116	1.056	.8470	.4487
REXBAT Projected	1.2	1.1	.9	.5

Section 5
CONCLUSIONS

5.1 Cost Factors

The efficiency of the STAGS program is one of its outstanding features and is well known to its many users. Thus, discussion here is directed to the impact to STAGS of incorporating the arbitrary grid finite difference capabilities of FIDAG.

As noted in Section 1, the FIDAG analysis of the grid is carried out separately from the STAGS analysis, and thus can be pro-rated over studies involving a variety of boundary conditions. From STAGS point of view, instead of generating finite difference coefficients internally, it simply reads them from the data base. Consequently, for typical engineering analysis, particularly in nonlinear studies, the cost difference between treating n uniformly placed nodes and n arbitrarily placed nodes is very minor. However, since the arbitrary placement generally allows adequate coverage of a structure with substantially fewer nodes, the use of arbitrary grids will generally result in a considerably lower analysis cost.

For a general grid on a shell, FIDAG is required to produce two coefficient matrices per control point: a low order one for membrane and a higher order one for bending. The results of this study indicate that the FIDAG cost per control point using 8 x 8 membrane and 12 x 12 bending matrices is about 0.08 seconds on the CDC 6600. With 6 x 6 membrane and 10 x 10 bending matrices the cost is about .05 seconds per control point.

5.2 Results

The sequence of tests presented in this report constitute a preliminary evaluation of the STAGS-FIDAG system. Bending and membrane problems have been treated separately and in combination via shell analysis. The results have

provided a basis for establishing suitable value ranges for the parameters involved, namely: (1) the number of control points per element, (2) the order of the numerical integration to be used (number of integration points per element) and (3) the order of finite difference approximation to be used. Further refinement of these ranges will be accomplished through broader problem experience.

In general applications, one will usually obtain conservative results with the system, i.e., predicted displacements will tend to be larger than the theoretically exact ones.

ACKNOWLEDGMENTS

The authors wish to thank Mr. Bo Almroth for providing the disc and cap test problems used in this study and his help in solving them. Thanks also go to Mr. William A. Loden and Dr. David Bushnell for providing the comparative solutions for the cap problems using the finite element program REXBAT and the shell of revolution program BOSOR.

Section 6

REFERENCES

1. Almroth, B. O., F. A. Brogan, and G. M. Stanley, "Structural Analysis of General Shells," Vol. 3 (User Instructions for STAGSC) LMSC Report, Lockheed Missiles & Space Company, Inc., Palo Alto, Calif. (Dec 1975), 230 pp

2. Bushnell, D., "Stress, Buckling and Vibration of Hybrid Bodies of Revolution," Proc. AIAA/ASME/SAE 17th Structures, Structural Dynamics and Materials Conf., 1976, pp 16-38

3. Clough, R. W. and C. A. Felippa, "A Refined Quadrilateral Element for the Analysis of Plate Bending," Proc. 2nd Conf. on Matrix Meth. in Structural Mech., AFFDL-TR-68-150, 1968, pp 399-440

4. Hermann, L R., "A Bending Analysis for Plates," Proc. Conf. Matrix Methods in Structural Mechanics, AFFDL-TR-66-80, Wright-Patterson Air Force Base, Dayton, Ohio (1966) pp 577-602

5. Jensen, P. S., "FIDAG, A Program for Finite Difference Interpolation on an Arbitrary Grid," Report LMSC-D502136, Lockheed Palo Alto Research Laboratory, April 1976, 65 pp

6. Jensen, P. S., "Solution of Partial Differential Equations by Arbitrary Grid Finite Difference Methods," Air Force report AFOSR TR-72-1301, AD 746 045 (July 1973) 54 pp

7. Jensen, P. S. and C. A. Felippa, "Variable Grid Finite Difference-Element Solution of Elliptic Partial Differential Equations," Air Force report AFOSR TR-74-1036, AD 78201 (May 1974) 54 pp

8. Loden, W. A. and Stearns, L. E., "What is This Thing Called 'REXBAT'?" LMSC Report D457042, Lockheed Research Laboratory, Palo Alto, Calif., May 1975

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER AFOSR - TR - 76 - [redacted] 1302	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle) 6 ENGINEERING ANALYSIS USING ARBITRARY GRID FINITE DIFFERENCE TECHNIQUES.		9 5. TYPE OF REPORT & PERIOD COVERED Final rept. 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) 10 Paul S. Jensen Frank A. Brogan		15 8. CONTRACT OR GRANT NUMBER(s) F44620-71-C-0109
9. PERFORMING ORGANIZATION NAME AND ADDRESS Lockheed Palo Alto Research Laboratory 3251 Hanover Street Palo Alto, California 94304		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS 61102F 2304/A3
11. CONTROLLING OFFICE NAME AND ADDRESS Air Force Office of Scientific Research (NM) Bldg 410, Bolling Air Force Base Washington, D. C. 20032	11	12. REPORT DATE 3 Aug[redacted]76 13. NUMBER OF PAGES 40
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office) 16 2304 17 A3 12 44p.		15. SECURITY CLASS. (of this report) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited.

18 AFOSR 19 TR-76-1302

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

14 LMSC/D507700

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

Eigenvalues
Eigenvectors
Generalized Eigenproblem
Symmetric
Sparse Matrices
Modes
Frequencies

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this report, several experiments on the analysis of shell structures using the STAGS-FIDAG computer program combination are described. STAGS (STructural Analysis of General Shells) is a general engineering analysis computer program particularly useful for nonlinear problems and FIDAG (Finite Differences on Arbitrary Grids) is a general program for finite difference interpolation on arbitrary grids.

DD FORM 1 JAN 73 1473 EDITION OF 1 NOV 65 IS OBSOLETE

UNCLASSIFIED

SECURITY CLASSIFICATION OF THIS PAGE (When Data Entered)

210118